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A STUDY OF REDUNDANT CONSTRAINTS IN LINEAR PROGRAMMING MODELS

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ABSTRACT

Linear Programming is a mathematical technique to help plan and to achieve the best outcome. It will make decisions relative to allocate resources and find the minimum or maximum value of the objective. While formulating a linear programming model, decision makers often tend to include all the possible constraints and variables although some of them may not be binding at the optimal solution. In fact, LP models almost contain a significant number of redundant constraints and variables. Therefore it is worthwhile to devote some efforts in solving for considerable reduction in the size of the problem. In this aspect, this paper proposes to identify which constraints and variables most likely to be tight at optimality.

Keywords - *linear programming models, redundant constraints, variables decrease method.*

I. INTRODUCTION

Linear programming models consist of an objective function which is to be maximized or minimized subject to a set of constraints. Many researchers have proposed different algorithms to solve linear programming models. The computational complexity of any linear programming model depends on the number of constraints and variables. In solving linear programming models, it is acknowledged that redundancies do exist in most of the practical linear programming models.

Telgen[2] first defined strongly and weakly redundant constraints. Boneh[3] first defined absolute and relative redundancy. Boneh and caron[4] first discussed weakly and strongly redundant constraints. Many researchers have proposed different algorithms to identify the redundancies and removed them to get a reduced model for linear programming. Thompson [5] use techniques for removing nonbinding constraints. Paulrajet al [6] proposed a heuristic method to identify redundant constraints by using the intercept matrix of constraints of a linear programming problem. Paulraj and Sumathi[7] introduced a comparative study of redundant constraints identification methods in linear programming models. Sumathi and Paulraj[9] submitted a new approach proposed for reducing time and moredatamanipulation by selecting a restrictive constraint in linear programming models to identify the redundant constraints.

II. DEFINITION AND CLASSIFICATION OF REDUNDANT CONSTRAINT

Redundant Constraints
A redundant constraint is a constraint that can be removed froma system oflinear constraintswithout changing the feasible region [7].

Binding Constraint
Binding constraint is the one which passes through the optimal solution point. It is alsocalleda relevant constraint.

Consider the following system of m nonnegative linear inequality constraints and $n$variables$m \geq n$:
$AX \leq b$, X≥0(2.1)

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where $A\in R^{m+n}$, $b\in R^m$, $X\in R^n$

Let $A_i X\le b_i$ be the i^{th} constraint of the system (2.1) and let $S=\{X\in R^n / A_i X\le b, X\ge 0\}$, be the feasible region associated with system (2.1).

Let $S_k=\{ X\in R^n/A_iX\le b_i, X\ge 0 i\ne k\}$ be the feasible region associated with the system of equations $A_i X\le b_i, i=1,\dots m, i\ne k$. The k^{th} constraint $A_k X\le b_i,(1\le k\le m)$ is redundant for the system(2.1)if and only if $S=S_k$.

III. ALGORITHM

This method [10]identifies the relevant set of constraints which finds out the optimal solution for the linear programming models.

The steps of the algorithm are as follow:

1. Construct a matrix of intercepts of all the decision variables formed by each of the resource constraints along the respective coordinate axis

$\Theta ji= \dfrac{b_i}{a_{ij}}$, $a_{ij} > 0$, i = 1,2…m and j = 1,2,…,n.

Find the min {ϴji} = βj in each row j.

2. Set L = {i : min $_{\text{atleast one } j}$ {ϴji}U<{ϴji}< maxβj}, for a j}, L is theindex of the selected set of constraints and assumes it contains p numberof constraints, p≤m.

3. Set J = {m + j, j = 1,2,3,…,n}

4. Find $\alpha_K= \dfrac{a_k c}{\|a_k\|}$, K ϵ L **U** J.

5. Set q = 1.

Solve the relaxed problem

$$\text{Maximize } Z = C^T X$$

Subject to the constraints

$arx \le br asx \le bs$

$x \ge 0$,

where the r^{th} and s^{th} constraints are selected by r= $(max_k a_k)$ and s= arg$(min_k a_k)$, provided αk>0. Let the solution obtained, be x (q).Check for violated constraints. If none is found, stop. Since x(q) is the optimal solution. Otherwise

6. Take l = arg(max αk)of the violated constraints if q is odd or Takel = arg(min αk)of the violated constraints if q is even. Set q=q+1.Appendthe violated constraint corresponding to index l to the relaxed problem andgo to step 5.

IV. VARIABLE DECREASE METHOD

In this section the use of variable decrease method [8] for linear programming model involving two variables and then, it will extend the algorithm involving 'n' variables.

Consider the following linear programming model involving two variables ((M) denotes given linear programming model)

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(M) Maximize z = $\sum_{j=1}^{2} c_j x_j$

Subject to

$$\sum_{j=1}^{2} a_{ij} \leq b_i \, i = 1, 2, 3, \ldots m.$$

$x_j \geq 0, j = 1, 2$

where $c_j \geq 0$, $a_{ij} \geq 0$, $b_i \geq 0 \forall$ i and j.

Algorithm

1. Find the minimum of the maximum possible integral values of x_1 and x_2 . Say U which corresponds to x_1 $[k]$ denotes the largest integral valuewhich is ≤ or = k.

Now, maximum possible integral value of $x_j$in the model (M), x^0_j is given by

x^0_j=Minimum of $\left\{ \left| \frac{b_i}{a_{ij}} \right|, i = 1,2,3,,,\ldots, n \right\}, j = 1, 2$

2.z(M) denotes the optimal value of z in the problem (M).

Consider the following(EM):

(EM): Find the value of z and the integral values of x_1 and x_2 such that

Maximize $\left\{ \begin{array}{c} z = \sum_{j=1}^{2} c_j x_j \\ (x_1, x_2) : \square\, P or Q \end{array} \right\}$

Where

P = $\left\{ \begin{array}{c} (x_1, x_2) : x_1 = \{1, 2,, \ldots U\} \\ x_2 = minimum \left\{ \begin{array}{c} \left[\frac{b_i - a_{i1} x_1}{a_{i2}} \right] \\ i = 1, 2, 3, \ldots, m \end{array} \right\} \end{array} \right\}$

Where U is the maximum possible integral value of x_1 and

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$$Q = \left\{ \begin{array}{c} (x_1, x_2) : x_2 = \{1,2,,\dots V\} \\ x_1 = minimum \left\{ \begin{array}{c} \left[\dfrac{b_i - a_{i2} x_2}{a_{i2}} \right] \\ i = 1,2,3,\dots,m \end{array} \right\} \end{array} \right\}$$

Where V is the maximum possible integral value of x_2 and

Now, it will prove a relationship between z and z*.

Solve the following equivalent model (EM) of the model (M). Let (x_1^0, x_2^0, z^0) be a solution of the model (EM).

4. It is easy to prove that (x_1^0, x_2^0, z^0) is a solution of the model(M) ⇔ (x_1^0, x_2^0, z^0) is a solution of the model (EM).

Algorithm II (for n variables)

1. Find the minimum of the maximum possibl eintegral values of $x_j = 1,2,3,\dots.n$ in the model (P). Say U_r which corresponds to x_r .

2. For each $\acute{x}_r$ _ {0,1,2,...,Ur} , we have a linear programming model (P($\acute{x}_r$))involving n-1 variablesx_j, j=1,2,…,n and j≠ r
where

(P($\acute{x}_r$))Maximize z = $c_r \acute{x}_r + \sum_{\substack{j=1 \\ j \neq r}}^{n} c_j x_j$

subject to

$$\sum_{\substack{j=1 \\ j \neq r}}^{n} a_{ij} x_j \leq b_i - a_r \acute{x}_r, i = 1,2,3,\dots.m$$

$$x_j \geq 0, j = 1,2,3,\dots n, j \neq r$$.

3. Find the minimum of the maximum possible integral values of x_j, j = $1,2,3,\dots.n$ and j ≠ r in the model (P ($\acute{x}_r$)), x_r= {0,1,2,...,U$_r$} . Say U_t which corresponds to x_t .

4. For each $\acute{x}_t$ ∊ {0,1,2,...,U$_t$} , we have a linear programming model(P($\acute{x}_r$, $\acute{x}_t$)) involving n-2 variables x_j, j =1,2,3.. ,n and j ≠ r, t
where

(P($\acute{x}_r, \acute{x}_t$))Maximize z = $c_r \acute{x}_r + c_r \acute{x}_t + \sum_{\substack{j=1 \\ j \neq r,t}}^{n} c_j x_j$

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subject to

$$\sum_{\substack{j=1 \\ j\neq r}}^{n} a_{ij} x_j \leq b_i - a_r \acute{x}_r - a_t \acute{x}_t, i=1,2,3,\ldots m$$

$$x_j \geq 0, j=1,2,3,\ldots n, j\neq r,t$$

5. Do Step 3 and 4 till a set of linear programming models (P($\acute{x}_r$, $\acute{x}_t$,..., $\acute{x}_k$)). $\acute{x}_k \in \{0,1,2,, U_k\}$ involving two variables is obtained.

6. Solve the set of linear programming models (P($\acute{x}_r$, $\acute{x}_t$,..., $\acute{x}_k$)) $\acute{x}_k \in \{0,1,2,, U_k\}$ individually by the algorithm I.

7. Find the maximum of z(P($\acute{x}_r$, $\acute{x}_t$,..., $\acute{x}_k$)) $\acute{x}_k \in \{0,1,2,, U_k\}$. Say z^0 which corresponds to the problem (P($\acute{x}_r$, $\acute{x}_t$,..., $\acute{x}_k$)) where t ={0,1,2,...,U_k} .

8. The solution to the problem (P) is z (P)=z^0, x'_k = t and solution of the model x_j ¿ $\acute{x}_j$, for all j.

V. CONCLUSION

The computational complexity of any linear programming model depends on the number of constraints and variables of the linear programming model. In this paper, a novel approach is used to identify the redundant constraints and variables. Also, discussed to select the constraints from given constraints and to solve the linear programming models. The variable decrease method is much useful for the decision makers. Computations are very simple and easy.It maintains feasibility as it moves from one vertex to another vertex in the feasible region

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